FRACTAL ANALYSIS OF JOINT SPACING AND LANDFORM FEATURES ON THE DARTMOOR GRANITE

A .J.GERRARD AND J. EHLEN

Gerrard, A. J., and Ehlen, J., Fractal analysis of joint spacing and landform features on the Dartmoor Granite. *Geoscience in south-west England,* **9**, 300-303.

Fractal analysis provides a method of quantifying the spatial and scaling properties of geometric data as a function of scale of spatial distribution. Many geological phenomena are scale invariant and fractal. This paper examines the fractal dimensions of one- and two-dimensional joint patterns within the Dartmoor granite, obtained by box-counting and Cantor's Dust methods, and compares these with dimensions calculated for drainage patterns, lineations and tor distribution. A good similarity between the fractal dimensions was obtained. Those for vertical joint spacings varied from 0.70 to 0.84, for lineations, drainage patterns and tor locations dimensions varied from 1.63 to 1.69 and for joint spacings on large rock pavements, values varied from 1.43 to 1.58. This similarity suggests that the patterns analysed are scale invariant over scales ranging from 1:1 to 1:50,000 and is further evidence for the uniform structural control of landscapes and individual features over these spatial scales.

A.J. Gerrard, School of Geography and Environmental Sciences, The University of Birmingham, Edgbaston, Birmingham B15 2TT
J. Ehlen, US. Army Topographic Engineering Center, 7701 Telegraph Road, Alexandria, Virginia 22315-3864

INTRODUCTION

The observation by Mandelbrot (1982), of the existence of a `Geometry of Nature' or fractal geometry, led to a new way of analysing a great variety of natural shapes and patterns. A fractal is a geometrical object whose fractal dimension is not an integer and is greater than the object's topological dimension (Barton *et al.,* 1991). Thus, a two-dimensional pattern should have a fractal dimension between 1.0 and 2.0. An object or feature is fractal if it is self-similar or scale invariant, if it is non-differentiable, and if it follows a power-law distribution. Geological patterns and landform shapes and patterns are obvious 'Geometries of Nature' that can be analysed in such a way. The use of fractal geometry offers intriguing possibilities for assessing the relationships between geological structures and landform patterns.

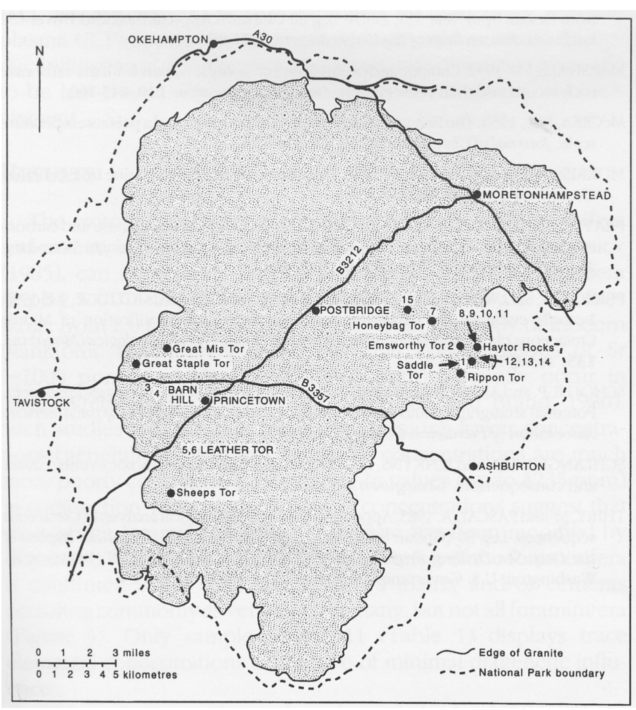

Figure 1. *Locations of the rock pavements analysed in the study*

Rock Pavement	Grid Reference	Size (sq. metres)
1 Saddle Tor	SX 752764	40.8
2 Emsworthy Tor	SX 751767	9.5
3 Barnhill no. 1	SX 532747	36.9
4 Barnhill no. 2	SX 532746	14.5
5 Leather Tor no. 1	SX 563697	25.6
6 Leather Tor no. 2	SX 563698	10.5
7 Honeybag Tor	SX 729787	35.8
8 Haytor no. 1	SX 758770	31.5
9 Haytor no. 2	SX 758770	35.4
10 Haytor no. 3	SX 758771	42.2
11 Haytor no. 4	SX 758769	31.2
12 Haytor Vale no. 1	SX 764766	18.9
13 Haytor Vale no. 2	SX 764765	8.3
14 Haytor Vale no. 3	SX 764766	7.2
15 Widecombe Manor	SX 716779	22.4

Table 1. *Rock pavements analysed and their respective sizes.*

Fractal analysis has been applied to a variety of geological phenomena but only to a limited extent to the spatial distribution of rock fractures. Analysis has been conducted on three types of fracture data: fracture spacing along a line sample (line survey data), fracture-trace maps (including lineation maps derived from remotely-sensed imagery) and pavement data. The majority of studies have used fracture-trace maps. Probably the first study reported in the literature was that of Barton and Larsen (1985) who obtained fractal dimensions for fracture patterns on pavements in a Miocene volcanic tuff unit at Yucca Mountain, Nevada. More recent studies, including analysis of fault patterns, have been those of Chiles (1985), Aviles *et al.* (1987), Hirata (1989), Kojima *et al.* (1989) and Velde *et al.* (1990). There have been a number of geomorphological studies involving fractal analysis but very few have attempted to examine geological patterns and landform patterns together. Dartmoor provides a good opportunity for examining such relationships. Relationships between the spacing and orientation of joints in the Dartmoor granite and landform features have been generally well established (e.g. Ehlen, 1989, 1991, 1994; Gerrard, 1974, 1978) but it has always been difficult to compare data obtained at different spatial scales. Fractal geometry, with its scale-invariant characteristic, would seem to offer a way around this problem. The results of such an analysis are presented here.

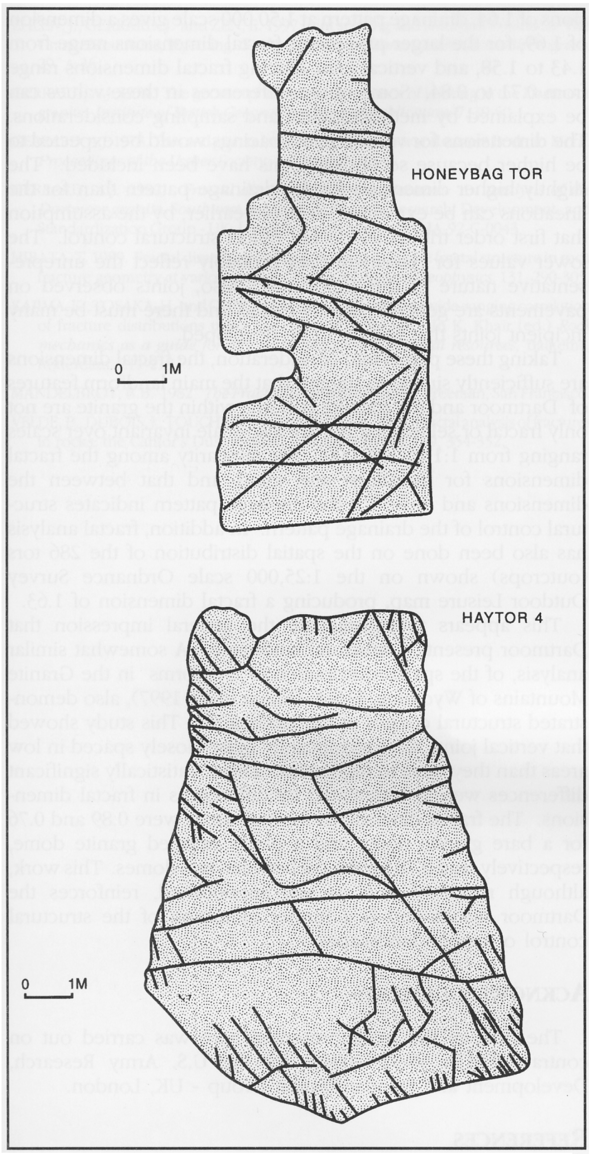

Figure 2. *Joint networks exposed on two of the granite pavements*

METHODOLOGY

Air photo analysis

Lineations were delineated on air photos at scales of 1:24,000 and 1:50,000. Lineations are defined as natural linear features on the earth's surface characterised by tonal changes in soil, linear vegetation patterns, topographic alignments and straight stream segments, among other features, which often represent the influence of geological control such as zones of joints or small faults. Lineations may be comparable to the long, usually open vertical or steeply dipping joints (primary joints) that control outcrop shape and possibly landscape evolution. There are approximately 1,500 lineaments on the 1:50,000 scale mosaic and 2,900 on the 1:24,000 scale mosaic. The lineation methodology is described in more detail in Ehlen (1998). The drainage network at a scale of 1:50,000 was also delineated on both scales of imagery.

Field methods

A modified version of the line survey was used to obtain spacings between vertical joints on outcrop faces (see Ehlen, 1998 for detail). The most accurate determination of fractal dimensions is obtained when the number of joints observed is large (a minimum of 100 joints is the usual rule of thumb; Barton 1993). This requirement restricted the number of outcrops on Dartmoor that could be used because vertical joints are typically very widely spaced and few tors are large enough to provide an adequate sample. The locations which provided the greatest number of joints in a single traverse were Great Mis Tor, Haytor, Lower Dunna Goat and Emsworthy Rocks.

Outcrops of granite, in the form of tors, are common on Dartmoor but horizontal outcrops or pavements are rare and small in individual extent. Only fifteen pavements were found that were thought worthy of study. These are located in Figure 1. They vary greatly in size, the largest being Haytor 3, with an area of 42.2 m^2 (Table 1). The pavements are clearly larger than this because they can be traced beneath a thin vegetation mat, but it was not feasible to expose larger pavements in the National Park. Some of the pavements are extremely small and, as will be discussed later, are probably too small to provide an adequate representation of the joint network.

Fractal geometry

The fractal dimensions for lineation overlays from the air photo mosaics and for pavement data were obtained using the grid- or box-counting method. This method is robust and straightforward and is the procedure used in previous studies (e.g. Barton and Larsen, 1985; Barton *et al.,* 1991). A sequence of grids, each with a different cell size, representing the different scales, is laid over the overlays or pavement and the number of cells or boxes containing segments of lineations or joints was counted. These data were plotted against box size in log-log space to determine the fractal dimension using the following equation, where N = number of filled boxes; δ = box size; and D = fractal dimension:

$$N(\delta) = 1/\delta^D$$

The most accurate calculation of the fractal dimension is obtained when the minimum number of cells is occupied for each cell size. This may be achieved by rotating the grid relative to the data until, for each cell size, the minimum number of cells is occupied. In general, for the pavement data, seven rotations were used, but the fractal dimensions for differing numbers of rotations varied only at the second decimal place (Gerrard, 1992). It was originally thought that box counting with rotation could be done physically on the pavements in the field. Thus, a portable mesh was laid on the pavement, after cleaning, and box counting undertaken. But this proved very time consuming, was possibly inaccurate, and did not result in a map of the joint network. As a result, each joint network was mapped, with the aid of the grid, and the fractal dimensions calculated from the joint map. Maps of two of the larger pavements are shown in Figure 2.

The vertical joint spacings from the line surveys were plotted on graph paper at a scale of 1:10 and, as the data are Cantor's Dusts, the fractal dimension was obtained as described by Velde *et al.* (1990). The grid or box-counting method, described above, was used to obtain the data. The fractal dimensions for these data were obtained from the following equation where P = proportion of filled boxes (n/N); δ = box size and D = fractal dimension:

$$P = \delta^{1-D}$$

		Fractal Dimension	**R^2 (%)**
Outcrops	Great Mis Tor	0.78	99.1
	Haytor	0.81	99.1
	Haytor	0.7	99.1
	Lower Dunna Goat	0.84	98.4
	Haytor	0.76	99.8
	Emsworthy Rocks	0.71	99.7
Lineations	1:24,000 scale	1.64	99.9
	1:50,000 scale	1.64	99.4
Drainage	1:50,000 scale	1.69	99.6

Table 2. *Vertical joint and lineation fractal dimensions (from Ehlen, 1998)*

Pavement	Fractal Dimension	R2 (%)
Bamhill no. 1	1.58	99.74
Leather Tor no. 1	1.58	99.71
Saddle Tor	1.55	99.16
Haytor no. 4	1.45	99.8
Haytor Vale no. 1	1.43	99.15
Barnhill no. 2	1.35	99.67
Leather Tor no. 2	1.31	99.69
Honeybag Tor	1.29	99.4
Emsworthy Tor	1.2	99.13
Haytor no. 1	1.16	99.15
Haytor Vale no. 3	1.15	86.49
Haytor no. 3	1.14	99.6
Haytor Vale no. 2	1.13	98.41
Haytor no. 2	0.9	99.8
Widecombe Manor	0.86	99.1

Table 3. *Fractal dimensions of joints on rock pavements and goodness of fit.*

RESULTS

The fractal dimensions for vertical joints in outcrops, lineations and drainage patterns are shown in Table 2. The mean fractal dimension for the vertical joints is 0.77 with a range of 0.70 to 0.84. The fractal dimensions for the lineations are the same at both image scales but lower than the dimensions for the vertical joints. The fractal dimension for the drainage network is slightly higher, although very similar, to those of the lineations. The higher dimension for the drainage pattern relative to the lineation pattern is to be expected because there appears to be no, or little, joint control in the first order tributaries (Gerrard, 1974). The lower dimensions for lineations and drainage pattern, compared to outcrop data, would also be expected because lineations are comparable to the less dense primary joints whereas the field data comprise secondary joints, as well as primary joints. Secondary joints are short, usually local in extent, and typically terminate against primary joints.

The fractal dimensions for the pavement data are more variable (Table 3). Three main groupings can be discerned. Group 1, comprising Saddle Tor, Barnhill no. 1, Leather Tor no. 1, Haytor no. 4 and Haytor Vale no. 1, with dimensions ranging from 1.43 to 1.58; Group 2, comprising Emsworthy Tor, Barnhill no. 2, Leather Tor no. 2, Honeybag Tor, Haytor nos. 1 and 2, and Haytor Vale nos. 2 and 3, with dimensions ranging from 1.13 to 1.31 and Group 3, comprising Haytor no. 2 and Widecombe Manor, with dimensions less than 1. It is Group 1 pavements which possess fractal dimensions within the range presented by Barton (1993). This group tends to include the larger pavements but not exclusively so. The fractal dimension by the box-counting method is an estimate of how well the particular phenomena under investigation, in this case joints, fill the object concerned, in this case pavements. Although not directly an indication of joint density there is little doubt that higher density networks fill the available space more completely. Also, the spatial relations between joints in the network will affect the fractal dimension, that is whether joints are evenly spaced, randomly spaced or are clustered. There is some justification for thinking that joint distribution tends to be more clustered than uniform. If this is so, a small pavement area might not represent the real distribution of joints and would tend to produce a lower fractal dimension. In many cases the joints revealed on the pavement are a reflection of random exposure and the fractal dimension might thus be slightly misleading. However, irrespective of the value of the fractal dimensions, all the patterns examined here are fractal and self-similar or scale invarant.

CONCLUSIONS

The fractal dimensions of the various patterns can be summarised as follows. Lineations at both scales have dimensions of 1.64, drainage pattern at 1:50,000 scale gives a dimension of 1.69, for the larger pavements fractal dimensions range from 1.43 to 1.58, and vertical joint spacing fractal dimensions range from 0.71 to 0.84. Some of the differences in these values can be explained by methodological and sampling considerations. The dimensions for vertical joint spacings would be expected to be higher because secondary joints have been included. The slightly higher dimension for the drainage pattern than for the lineations can be explained, as noted earlier, by the assumption that first order tributaries do not reflect structural control. The lower values for pavement data probably reflect the unrepresentative nature of the exposures. Also, joints observed on pavements are generally tightly closed and there must be many incipient joints that remain to be developed.

Taking these points into consideration, the fractal dimensions are sufficiently similar to suggest that the main landform features of Dartmoor and the fracture patterns within the granite are not only fractal or self-similar but are also scale invariant over scales ranging from 1:1 to 1:50,000. The similarity among the fractal dimensions for lineations and joints and that between the dimensions and those for the drainage pattern indicates structural control of the drainage pattern. In addition, fractal analysis has also been done on the spatial distribution of the 286 tors (outcrops) shown on the 1:25,000 scale Ordnance Survey Outdoor Leisure map, producing a fractal dimension of 1.63.

This appears to substantiate the general impression that Dartmoor presents a structural landscape A somewhat similar analysis, of the spatial distribution of landforms in the Granite Mountains of Wyoming, U.S.A. (Ehlen *et al.,* 1997), also demonstrated structural control of the landscape. This study showed that vertical joints were significantly more closely spaced in low areas than they were in high areas. These statistically significant differences were substantiated by differences in fractal dimensions. The fractal dimensions for high areas were 0.89 and 0.76 for a bare granite dome and a partly stripped granite dome, respectively, and 0.76 for the col between two domes. This work, although much of it is as yet unpublished, reinforces the Dartmoor results and is a further indication of the structural control of granite landscapes.

ACKNOWLEDGEMENTS

The bulk of the work presented here was carried out on contract (DAJA 45-92-C-0043) for the U.S. Army Research, Development and Standardization Group - UK, London.

REFERENCES

AVILES, C.A., SCHOLZ, C.H. and BOATRIGHT, J. 1987. Fractal analysis applied to characteristic segments of the San Andreas Fault. *Journal of Geophysical Research,* **92**, B1, 331-344

BARTON, C.C. 1993. Fractal analysis of the scaling and spatial clustering of fractures. In *Fractals and their use in earth sciences,* Geological Society of America, Short Course Notes, Boston, 29-30 October, 1993, 171-243.

BARTON, C.C. and LARSEN, E. 1985. Fractal geometry of two-dimensional fracture networks at Yucca Mountain, southwest Nevada. In Stephannnson, O. (ed.) *Fundamentals of rock joints,* Proceedings of the International Symposium on Fundamentals of Rock Joints, Bjorkliden, Lapland, Sweden, Centek Publishers, Lulea, Sweden, 74-84.

BARTON, C.C., LAPOINTE, P.R. and MALINVERNO, A. 1991. *Fractals and their use in Earth Sciences,* Short Course Manual, Geological Society of America Annual meeting, San Diego, California, 1991.

CHILES, J.P. 1985. Fractal and geostatistical methods for modelling of a fracture network. *Mathematical Geology,* **20**, 631-654.

EHLEN, J. 1989. *Geomorphic, petrographic and structural relationships in the Dartmoor granite, Southwest England.* Unpublished PhD thesis, University of Birmingham, England.

EHLEN, J.1991. Significant geomorphic and petrographic relations with joint spacing in the Dartmoor granite. *Zeitschrift fur Geommphologie,* NS **35**, 425-438.

EHLEN, J.1994 Classification of Dartmoor tors. In Robinson, D.A. and Williams, R.B.G. (ed.) *Rock weathering and landform evolution, John* Wiley and Sons, Chichester, 393-412.

EHLEN, J.1998. A proposed method for characterizing fracture patterns in denied areas. *Geological Society of America, Reviews in Engineering Geology,* **XIII**, 151-163.

EHLEN, J., GERRARD. J. and ZEN, E. 1997 Joint spacing and landform evolution: The Granite Mountains, Wyoming. *Geological Society of America Abstracts with Program,* **29**, A-36.

GERRARD, A.J. 1974. The geomorphological importance of jointing in the Dartmoor granite. *Institute of British Geographers Special Publication*, **7**, 39-50.

GERRARD, A.J. 1978. Tors and granite landforms of Dartmoor and eastern Bodmin Moor. *Proceedings of the Ussher Society,* **4**, 204-210.

GERRARD, A.J. 1992. *Fractal dimensions of rock fractures. An analysis of the Dartmoor granite, Southwest England.* U.S. Army Research, Development, and Standardization Group - UK, London, report number DAJA-92-C-0043.

HIRATA, T. 1989. Fractal dimensions of fault systems in Japan: fractal structure in rock fracture geometry at various scales. *Pure and Applied Geophysics,* **131**, 290-305.

KAJIMA, K., TOSAKA, H. and OHNO, H. 1989. An approach to wide-ranging correlation of fracture distributions using the concept of fractals. In K. Khair (ed.) *Rock mechanics as a guide for efficient utilization of natural resources,* Balkema, Rotterdam, 211-218.

MANDELBROT, B.B. 1982. *The Fractal Geometry of Nature,* Freeman, San Francisco.

VELDE, B., DUBOIS, J., TOUCHARD, G. and BADRI, A. 1990. Fractal analysis of fractures in rocks: the Cantor's Dust method. *Tectonophysics,* **179**, 354-352.